Energy-time entanglement, Elements of Reality, and Local Realism

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The Franson interferometer, proposed in 1989 [J. D. Franson, *Phys. Rev. Lett.* **62**:2205–2208 (1989)], beautifully shows the counter-intuitive nature of light. The quantum description predicts sinusoidal interference for specific outcomes of the experiment, and these predictions can be verified in experiment. In the spirit of Einstein, Podolsky, and Rosen one may now ask if the quantum-mechanical description (of this setup) can be considered complete. This question will be answered in detail in this paper, by delineating the quite complicated relation between energy-time entanglement experiments and Einstein-Podolsky-Rosen (EPR) elements of reality. The mentioned sinusoidal interference pattern is the same as that giving a violation in the usual Bell experiment. Even so, depending on the precise requirements made on the local realist model, this can imply a) no violation, b) smaller violation than usual, or c) full violation of the appropriate statistical bound. Alternatives include a) using only the measurement outcomes as EPR elements of reality, b) using the emission time as EPR element of reality, c) using path realism, or d) using a modified setup. This paper discusses the nature of these alternatives and how to choose between them. The subtleties of this discussion needs to be taken into account when designing and setting up experiments intended to test local realism. Furthermore, these considerations are also important for quantum communication, for example in Bell-inequality-based quantum cryptography, especially when aiming for device independence.

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I. INTRODUCTION

In 1989 a new interferometric setup was proposed by J. D. Franson [1]. The main intent was to test the possibility of local realist models as a possible description, more complete than quantum mechanics. The sinusoidal interference obtained from the experiment when restricting to coincident events is larger than the bound from given by the Bell inequality [2]. But the selection of coincident events at the two sites introduces postselection into the data analysis. This need for postselection has been under discussion for some time [3–8], and this paper is intended to review the discussion and to provide some insight into the matter at hand. We will see that, depending on what is required from the tested model class, the appropriate inequality changes so that the same experimental outcomes in some cases do violate the Bell inequality as usual, and in some cases do not. Interestingly, the class of models that uses EPR elements of reality (and nothing more) falls between these two, and violates the Bell inequality at a lesser degree than in other setups.

The paper is organized as follows: the rest of the Introduction is devoted to background, in a level of detail that enables an in-depth discussion in what follows. Section II introduces the Franson interferometer and discusses effects of postselection. In Section III, the usual Bell inequality is re-established by adding path realism to the model class. Section IV concentrates on using only EPR elements of reality, giving a weaker inequality but nonetheless a violation of local realism, and Section V contains a few examples of modified experimental setups and their properties.

A central concept in this analysis is "EPR elements of reality" as proposed by Einstein, Podolsky, and Rosen (EPR) in 1935 [9]. The concept is well-known, but a brief repetition is in place. The setting is as follows: consider a (small) physical system on which we intend to measure position Q or momentum P. The physical measurement devices associated with these measurements are mutually exclusive, and furthermore, the quantum-mechanical description for this physical system tells us that the measurements Q and P do not commute. The standard way to interpret this is that the system does not possess the properties of position or momentum, only probabilities are possible to obtain from quantum mechanics.

What EPR ask in their paper is whether the quantum-mechanical description can be considered complete, or if it is possible to argue for another, more complete description. They use a combined system of two subsystems of the above type, in a combined state so that joint measurement of the positions gives the position mean $Q_1+Q_2=0$ and joint measurement of the two momenta give the momentum difference $P_1-P_2=0$. These two combinations are measurable at the same time, even though the individual positions and momenta are not, which means that it is possible to produce a joint state with these properties. Letting the two subsystems separate, usually very far, they consider individual measurement of position or momentum. The system is such that the position mean and momentum difference is preserved under the separation process, which means that if the position of one subsystem has been measured, the position of the remote subsystem can be predicted. Therefore, EPR argue, the position of the remote subsystem must exist as a property

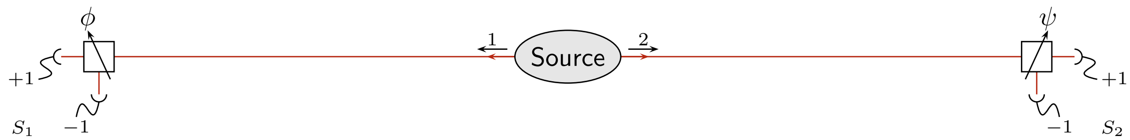

FIG. 1. The EPR-Bohm-Bell setup. The two systems are spin-1/2 systems, and the local measurements are made along a direction in space ϕ or ψ, respectively. The source is such that if the directions $\phi = \psi$, the outcomes $S_1+S_2 = 0$ with probability one.

of the subsystem. EPR write:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Likewise, if the momentum of one subsystem has been measured, the momentum of the remote subsystem can be predicted. In this case, the momentum of the remote subsystem must exist as a property of the subsystem. EPR continue to argue that *both* position and momentum must simultaneously exist as properties of the remote subsystem, otherwise

> ...the reality of P and Q depend[s] upon the process of measurement carried out on the first system, which does not disturb the second [remote] system in any way. No reasonable definition of reality could be expected to permit this.

The above definition of an "EPR element of reality" is the philosophical motivation for considering properties of a system as existing, independent of measurement. The possibility of remote prediction ("without in any way disturbing a system") enables the notion of EPR element of reality, and we will use this notion below to motivate existence of properties of the involved physical systems.

The EPR setup is fine for philosophical considerations, but there are more powerful tools that enable a statistical test of experimental data. The setup to be used was proposed by D. Bohm in the fifties [10] (see Fig. 1), and uses a system combined of two spin-1/2 subsystems in a total spin 0 state, so that the spins $S_1 + S_2 = 0$ when measured along equal directions $\phi = \psi$. The subsystems are allowed to separate and a spin measurement is made on one of the subsystems. The choice of measurement directions is a continuous choice instead of the dichotomic choice in the original EPR setup. When a measurement has been made on one subsystem, the result can be used to predict the result of a measurement on the remote subsystem along the same direction. And because the reality of the spin measurement result in the remote system cannot depend on the local choice, the spin along *any* direction is an EPR element of reality.

This was used in the celebrated Bell paper [2] where a statistical test was devised, in the form of an inequality that must be fulfilled by any mathematical model that is realist and local. Realism is here motivated by the spin being an EPR element of reality, and locality is motivated by the finite speed of light, or more specifically, because local measurement is made "without in any way disturbing" the remote system (the below formulation is adapted from [11]).

Theorem 1:—A local realist model has the following two properties:

a) Realism: *Outcomes are given by random variables (on a prob. space $(\Lambda, \mathcal{F}, P)$),*

$$S_i(\phi, \psi) : \Lambda \mapsto \{\pm 1\},$$

b) Locality: *Outcomes do not depend on the remote settings,*

$$S_1(\phi, \psi) = S_1(\phi), \quad S_2(\phi, \psi) = S_2(\psi).$$

The outcomes from a local realist model obey

$$\begin{aligned} &\Big|E\big(S_1(\alpha)S_2(\delta)\big) + E\big(S_1(\alpha)S_2(\beta)\big)\Big| \\ &\quad + \Big|E\big(S_1(\gamma)S_2(\beta)\big) - E\big(S_1(\gamma)S_2(\delta)\big)\Big| \leq 2. \end{aligned} \tag{1}$$

The last inequality is violated by the predictions of quantum mechanics, using a total spin zero state that gives the correlation

$$\big\langle s_1(\phi)s_2(\psi)\big\rangle = -\cos(\phi - \psi) \tag{2}$$

with $\phi - \psi$ being the angle between the two directions ϕ and ψ. Choosing the four directions $\pi/4$ apart in a plane in the order δ, α, β, γ, one obtains

$$\begin{aligned} &\Big|\big\langle s_1(\alpha)s_2(\delta)\big\rangle + \big\langle s_1(\alpha)s_2(\beta)\big\rangle\Big| \\ &\quad + \Big|\big\langle s_1(\gamma)s_2(\beta)\big\rangle - \big\langle s_1(\gamma)s_2(\delta)\big\rangle\Big| = 2\sqrt{2}, \end{aligned} \tag{3}$$

and this violates Ineq. (1). Therefore, the quantum-mechanical predictions cannot be obtained from a local realist model.

There is one problem that is present in most experiments done to test this inequality: inefficient detectors. This problem was first noticed by Pearle in 1970 [12], but has been treated in many subsequent papers. The problem is that one only obtains detections from a subset of the pairs in the experimental setup, the pairs that give

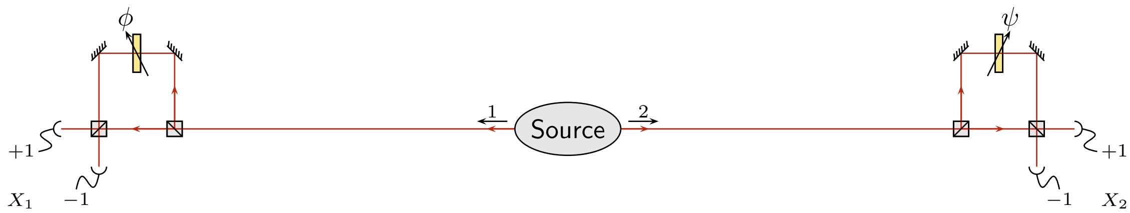

FIG. 2. The Franson setup. The source sends out time-correlated photons at unknown moments in time. These travel through unbalanced (but equal) Mach-Zehnder interferometer with variable phase delays ϕ and ψ. If the detections are coincident and $\phi + \psi = 0$, then $X_1 = X_2$ with probability one.

coincident detections. The correlation obtained from experiment is conditioned on coincident detection of two particles, one at each side. Taking this into account, the theorem needs to be modified as follows (adapted from [11]).

Theorem 2:—A local realist model with inefficiency has the following three properties:

a) Realism: *Outcomes are given by random variables on subsets of detection (in a prob. space* $(\Lambda, \mathcal{F}, P)$*),*

$$S_i(\phi, \psi) : \Lambda_{S_i,\phi,\psi} \mapsto \{\pm 1\},$$

b) Locality: *Outcomes and detections do not depend on the remote settings,*

$$\begin{aligned} S_1(\phi, \psi) = S_1(\phi) \text{ on } \Lambda_{S_1,\phi,\psi} = \Lambda_{S_1,\phi}, \\ S_2(\phi, \psi) = S_2(\psi) \text{ on } \Lambda_{S_2,\phi,\psi} = \Lambda_{S_2,\psi}, \end{aligned}$$

c) Efficiency: *There is a lower bound to the efficiencies,*

$$\eta = \min_{\substack{\text{settings} \\ \text{local sites}}} P\Big(\text{coincidence}\Big|\text{local detection}\Big).$$

The outcomes from a local realist model with inefficiency obey

$$\begin{aligned} &\Big|E\big(S_1(\alpha)S_2(\delta)\big|\text{coinc.}\big) + E\big(S_1(\alpha)S_2(\beta)\big|\text{coinc.}\big)\Big| \\ &+ \Big|E\big(S_1(\gamma)S_2(\beta)\big|\text{coinc.}\big) - E\big(S_1(\gamma)S_2(\delta)\big|\text{coinc.}\big)\Big| \\ &\qquad\qquad \leq \frac{4}{\eta} - 2. \end{aligned} \quad (4)$$

The effect is that the inequality is weakened by inefficient detectors, and is no longer violated by quantum mechanics at $\eta = 2\sqrt{2} - 2 \approx 82.83\%$. We will see effects and extensions of this below.

II. THE FRANSON INTERFEROMETER

In 1989 a new experimental setup was proposed by J. Franson [1] (see Fig. 2). The setup uses a source that emits time-correlated photons at unknown moments in time, and two unbalanced Mach-Zehnder interferometers. The interferometers should have a path difference that is large enough to prohibit first-order interference. Therefore, the probability is equal for a photon to emerge from each port of the final beamsplitter. But the interferometer path-differences should be as equal as possible; the path-difference difference (repetition intended) should be so small that the events of both photons "taking the long path" and both photons "taking the short path" are indistinguishable. Quotation marks are used here to remind the reader that photons are not particles, but quantum objects and as such, do not take a specific path. Given this indistinguability, since the emission time is unknown (and is a quantum variable in second quantization), there can be interference between two possibilities on the second beamsplitter.

There will be no interference if one photon "takes the long path" and the other "takes the short," because then, the emission time can be calculated easily as the early detection time minus the short path length divided by c. The emission time is known in this case. When both photons "take the same path," the emission time remains unknown, and this is what enables the interference. The interference is not visible as a change in output intensity, as in first-order interference, but instead in correlation of the outputs. Given coincident detection, if the total phase delay $\phi+\psi = 0$, then a photon emerging in the $+1$ channel on the left is always accompanied by a photon emerging in the $+1$ channel on the right, and the same for the -1 channels. Therefore, when a measurement has been made at one interferometer, the result can be used to predict what port the photon will emerge from at the remote interferometer, for $\phi + \psi = 0$. Since the local phase delay can be chosen freely, the output port along *any* direction is an EPR element of reality, when coincidence occurs. As a function of the total phase delay, we have

$$\big\langle x_1(\phi)x_2(\psi)\big|\text{coinc.}\big\rangle = \cos(\phi + \psi), \quad (5)$$

note the similarity to Eq. (2). It is now simple to obtain

$$\begin{aligned} &\Big|\big\langle x_1(\alpha)x_2(\delta)\big|\text{coinc.}\big\rangle + \big\langle x_1(\alpha)x_2(\beta)\big|\text{coinc.}\big\rangle\Big| \\ &\quad + \Big|\big\langle x_1(\gamma)x_2(\beta)\big|\text{coinc.}\big\rangle - \big\langle x_1(\gamma)x_2(\delta)\big|\text{coinc.}\big\rangle\Big| = 2\sqrt{2}, \end{aligned} \quad (6)$$

which exceeds the bound in Ineq. (1). The important question is now: does this setup violate local realism?

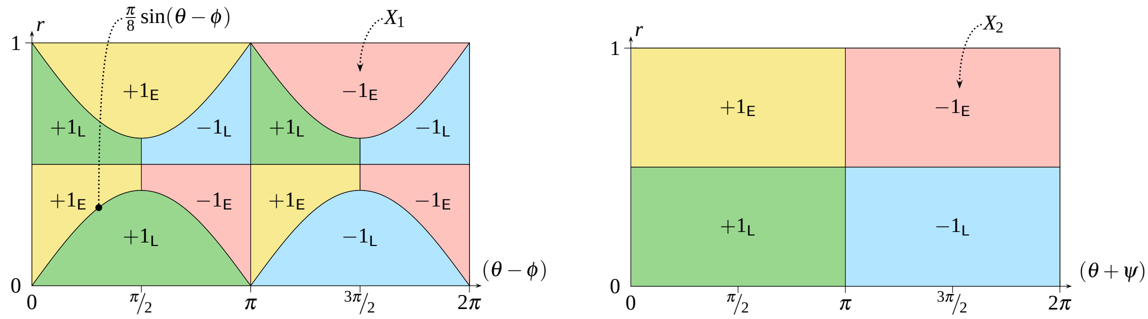

FIG. 3. The local hidden variable model of Ref. [4]. The hidden variable is a pair of numbers $\lambda = (\theta, r)$ evenly distributed over the rectangle $\Lambda = \{(\theta, r) : 0 \leq \theta < 2\pi, 0 \leq r < 1\}$. The outcomes are determined by the above graphs where detections are "early" (E) or "late" (L). This model reproduces the quantum predictions for the Franson interferometric setup, including those for the coincident detections.

The immediate answer is no. The problem is that there is a local realist model that gives the exact same predictions as quantum mechanics [4] (see Fig. 3). Since the model is of the type described in Theorem 1, it is strange that it seems to violate Ineq. (1). But it only *seems* to give a violation. The model does not violate the inequality; it is true that the model gives the correlation

$$E\big(x_1(\phi)x_2(\psi)\big|\text{coinc.}\big) = \cos(\phi+\psi), \tag{7}$$

but this is a *conditional* expectation, of the type used in Theorem 2. To reach this correlation we need to postselect coincident events, which returns only 50% of all events so that $\eta = 50\%$, well below the 82% bound where quantum-mechanical predictions are possible to reach with a local realist model.

Actually, this model uses a slightly different mechanism than inefficiency, it uses variable delays at the two sites, in which case the following theorem applies (for details see [13]).

Theorem 3:—A local realist model with delays has the properties a) and b) from Theorem 1, and

c) Delays: *Detections occur after local realist time delays,*

$$T_1(\phi) : \Lambda \mapsto \mathbb{R} \quad \text{and} \quad T_2(\psi) : \Lambda \mapsto \mathbb{R},$$

and usage of a coincidence window gives an apparent efficiency of

$$\eta = \min_{\substack{\text{settings}\\ \text{local sites}}} P\Big(\text{coincidence}\Big|\text{local detection}\Big).$$

The outcomes from a local realist model with delays obey

$$\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big) + E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big| + \Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big) - E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big| \leq \frac{6}{\eta} - 4. \tag{8}$$

In the interferometric setup under study, the delays are not continuously distributed but rather limited to two values: "delayed" and "not delayed," and a coincidence occurs only for equal delays on the two sides, corresponding to a coincidence window is smaller than the delay. In the above inequality, the bound for quantum-mechanical violation is $\eta = 3 - 3/\sqrt{2} \approx 87.87\%$, which is higher than the standard bound. The apparent efficiency is calculated as before and remains at 50% which is far from the efficiency needed for a violation.

Of course it is desirable to re-establish a violation. To do this one must first understand the basic reason for the failure of Theorems 1–3 to provide a usable test, in other words, the basic reason for the existence of the model in Fig. 3. It is tempting to blame only the postselection, but this is not the whole story. One should note that the theorems treat the individual sites as black boxes; feed them a setting and they give a (possibly delayed) ± 1 outcome, much like the setup of Fig. 1. We will see that taking more properties of the setup into account will enable a violation, and the remainder of this paper will discuss the possible ways to do this.

III. PATH REALISM

The key ingredient of the local realist model above [4] is that the delay at one of the sites depend on the relation between the hidden variables (θ, r) and the local setting (ϕ or ψ) at the site. To avoid this, one possibility is to have the "path taken by the photon" as a realist property [5, 8], in essence requiring particle-like properties of the photons. In this case, a photon will encounter the first beamsplitter before the variable phase delay, so that the "decision" to "take the long path or the short path" must be independent of the phase delay setting of the interferometer—the *local* measurement chosen—in contrast to a standard Bell experiment where only independence from the *remote* measurement choice is required.

It is certainly possible to list path realism as an ex-

pected model property, and to test it. But it is important to note that, in this setup, path realism is very different from measurement-outcome realism. The outcomes are EPR elements of reality because they can be remotely predicted, and by locality they can be argued to exist independently of what remote measurement was made. On the other hand, the path taken is not an EPR element of reality because it cannot be remotely predicted. There is no measurement at one site that enables a path prediction for the remote site in the setup of Fig. 2.

One may attempt to argue for path realism by bringing in properties from classical physics into the picture [5], which would make models like that in Fig. 3 inconsistent. However, the question at hand is not if an experiment like the above can be described within classical physics; there is no doubt that this is not possible. Instead, the question is whether the quantum-mechanical model can be considered complete. It is entirely possible that our classical intuition fails us, while quantum mechanics still can be completed. The discussion on EPR-Bell arguments is an attempt to find precisely what minimal requirements are needed to give a contradiction with quantum predictions. This question cannot be answered if the model is required to obey the complicated requirements from classical physics.

Another argument for path realism [8] would be to appeal to local prediction, rather than prediction from the remote site as EPR do. By measuring in one path and finding the photon there, one can predict that it is not present in the other. Locality and spacelike separation between the paths could then be used as support for path realism, and one could even attempt to extend the notion of reality [8] by changing "without in any way disturbing a system" into "spacelike separation." The reason for this modification would be that using EPR requires that a photon—when detected in one path—is a different undisturbed system when predicted not to be present in the other path, which is clearly not the case. It is true that finding the photon in one position enables a prediction that it is not anywhere else. But this cannot be used as evidence of an underlying realist model. Prediction of properties of *one system* immediately after measurement merely suggests that the measurement is repeatable.

Furthermore, it is central in the EPR reasoning that the system that we predict properties for is unaffected by the measurement used for predictions. EPR could choose to remotely predict position or momentum and therefore conclude the simultaneous reality of both. However, photon path is only available through a measurement in the local interferometer, and such a measurement prevents the remote prediction of the interferometer output, since the interference is destroyed in the process. This means that we cannot conclude in the same way that path and interferometer output both simultaneously are realist properties. Even though it is claimed in Refs. [5, 8] that path must be a realist property independently of whether the path measurement is performed or not, this is clearly not supported by EPR-style reasoning.

If one chooses to use path realism, it should be pointed out that path realism on its own does not prohibit interference. First order interference is not prohibited, since the model could randomly select a path for the photon and send an "empty wave" through the other path, that registers any phase shift in that path. The phase shift difference can be used to determine through which output port to emit the photon from the second beamsplitter. Second order interference in the present setup is also not directly prohibited by path realism itself, because the same mechanism would work.

However, large second order interference will be prohibited by the combination of path realism and local realist interferometer output. These two together will enable Theorem 1 separately both for Late-Late coincidences (both photons "taking the long path") and Early-Early coincidences (both photons "taking the short path"), because the delays cannot depend on the local choice of phase delay settings. This re-establishes the bound on the whole set of coincidences [5].

Theorem 4:—A local realist model with path realism has the properties a) and b) from Theorem 1, and

c) Path realism: *The path taken is a realist property, and is setting-independent.*

The outcomes from a local realist model with path realism obey

$$\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big| + \Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)-E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big| \leq 2. \tag{9}$$

Of course, path realism requires intimate knowledge of the internals of the interferometer, for example, that there really are two distinct paths used. This means that path realism makes it difficult to argue for device independence, since the requirements used involve low-level properties of the devices. Also, an experiment meant to test the inequality needs to ensure that the "decision" to "take the long path or the short path" really is independent of the phase delay setting. This would be accomplished by space-like separation of the choice of phase delay and the "photon's path choice," that is, the event of "the photon passing the beamsplitter." Since the experimenters do not know the detection moments in advance, this must be done by switching the settings fast enough to ensure that a new random setting is chosen in-between these two events. This is in contrast to a Bell experiment where the setting choices only need to be chosen at spacelike separation from the emission and each other, which is a much less demanding task.

Using path realism does have benefits, since postselection has no negative effects on the bound. The appropriate inequality has the standard bound (9) and is violated by the quantum prediction (5). There is no local realist model with path realism that gives the probabilities predicted by quantum mechanics for the Franson interferometer.

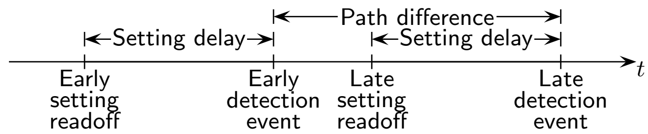

FIG. 4. Two settings influence the outcome, one for the early detection, and one for the late detection. The setting delay is the time that it takes for the information to travel from the phase delay to the detectors, via the light path.

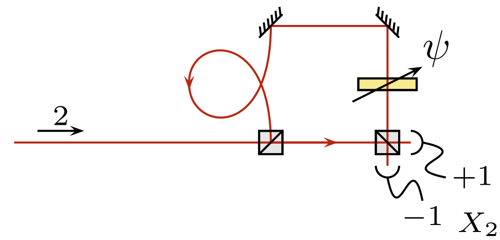

FIG. 5. To ensure the events occur in the order indicated in Fig. 4, three things can be done: a) making the path length difference large, b) placing the phase delay late in the interferometer, and c) placing the detectors close to the interferometer.

IV. LOCAL REALISM ONLY

It is now interesting to ask whether it is possible to obtain a violation using *only* EPR elements of reality. Obviously, local realist measurement outcomes is not enough. It *is* possible to establish another EPR element of reality in this setup, but it is slightly different than the path realism used above. By removing the first beamsplitter at one site (or the entire interferometer), the detection time can be used to calculate the time of emission of the photon pair. And this enables prediction of the emission moment for the remote photon, without in any way disturbing it. Therefore, *the moment of emission is an EPR element of reality.* Which means that a local realist model must be similar to the one in Fig. 3, in that it needs to specify the delay: whether a detection is "early" or "late." This is not the same as specifying the path because it does not require particle-like behaviour of the quantum object, so that the photon "takes a path." The only requirement is that the detection is delayed or not by the interferometric setup, treating the interferometer like a black box just as it is in the original Bohm-Bell setup. The important observation here is that the detection moment is also an element of reality and must be present in any local realist model.

In this situation, the settings ϕ and ψ can still influence whether a delay occurs or not. The event that needs to be spacelike separated from the setting choices is no longer "passing the first beamsplitter" (a point in time which here is EPR element of reality), it is instead the event of detection. This is because the detection is the event that gets delayed (or not) in the local realist model. The problem is that detection takes place after the photon has (or could have) passed the phase delay, so it seems impossible to have spacelike separation between setting choice and detection event.

Fortunately, there are two possible detection events, one early and one late. This means that it is possible to have two different settings for the two detection events: one at the "early-setting readoff" event for the early detection, and one at the "late-setting readoff" event for the late detection (see Fig. 4). The choice of the early setting cannot be spacelike separated from any of the possible detection events since they are both inside (or on) the forward light cone with respect to it. The choice of the late setting, however, can be spacelike separated or even inside the forward light cone of the (possible) early detection event. To make this possible, one needs to make the path difference of the interferometer longer than the distance from the phase delay to the detectors. In other words, one needs to make the path difference large, to place the phase delay as late as possible in the interferometer, and to have the detectors close to the interferometer (see Fig. 5).

When this is the done, the early setting can still be used by a (hypothetical) model to delay the detection. But the late setting cannot undo the delay, which the model would have to do for some combinations of hidden variables and local settings. This is because the early detection event would already have taken place when the late-setting choice is made. So, while there is no bound (except the trivial) for the outcomes in the early coincidences, Theorem 1 applies for the late coincidences, and this gives the following theorem [4].

Theorem 5:—A local realist model with long realist delays has the properties a) and b) from Theorem 1, and

c) Long realist time-delays: *The delay is a realist property (as in Theorem 3c), and is long enough to ensure that the setting relevant for the late detection cannot undo the delay.*

The outcomes from a local realist model with long realist delays obey

$$\begin{aligned}&\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)-E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big|\leq 3.\end{aligned}\tag{10}$$

The bound in (10) is 3; this is the mean value of the trivial bound 4 for the early coincidences and the bound 2 for the late coincidences. Unfortunately, this is larger than the maximal quantum prediction $2\sqrt{2}$. To establish a better bound we need to use so-called "chained" Bell inequalities [14] with more terms, and this gives the following theorem [4].

Theorem 6:—The outcomes from a local realist model

with long realist delays (as specified in Theorem 5) obey

$$\begin{aligned}&\Big|E\big(X_1(\alpha)X_2(\varphi)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)+E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\epsilon)X_2(\delta)\big|\text{coinc.}\big)-E\big(X_1(\epsilon)X_2(\varphi)\big|\text{coinc.}\big)\Big|\leq 5.\end{aligned}\tag{11}$$

Again the bound is the mean value of the trivial bound 6 for the early coincidences and the chained-Bell bound 4 for the late coincidences. While the postselection does have an effect on the bound, the quantum-mechanical prediction still gives a violation; choosing the six directions $\pi/6$ apart in a plane in the order φ, α, β, γ, δ, ϵ, will yield the quantum prediction

$$\begin{aligned}&\Big|\big\langle x_1(\alpha)x_2(\varphi)\big|\text{coinc.}\big\rangle+\big\langle x_1(\alpha)x_2(\beta)\big|\text{coinc.}\big\rangle\Big|\\&+\Big|\big\langle x_1(\gamma)x_2(\beta)\big|\text{coinc.}\big\rangle+\big\langle x_1(\gamma)x_2(\delta)\big|\text{coinc.}\big\rangle\Big|\\&+\Big|\big\langle x_1(\epsilon)x_2(\delta)\big|\text{coinc.}\big\rangle-\big\langle x_1(\epsilon)x_2(\varphi)\big|\text{coinc.}\big\rangle\Big|\\&=6\cos\tfrac{\pi}{6}\approx 5.196,\end{aligned}\tag{12}$$

There is no local realist model with setting-independent delays that gives the probabilities predicted by quantum mechanics. An experiment to test this is more difficult than an experiment to test Theorem 4, since it is less resilient to noise because the quantum violation is lower. The visibility needs to be 96.2% which is demanding. This can be improved somewhat by adding terms to the Bell inequality (extending the "chain," see Table I), but the best possible bound is for 10 terms, at 94.6%.

Here it is easier to argue for device independence since one does not rely on internal properties of the interferometer. It is, of course, still important to verify that the delays do occur at equal probability, but it is no longer needed to verify the existence of two distinct paths. Similarly as before, an experiment meant to test the inequality needs to ensure that the possible early detection event really is independent of the late phase delay choice. And again, since the experimenters do not know the detection moments in advance, this must be done by switching the settings fast enough to ensure that a new random setting is chosen in-between the early detection event and the late-setting readoff. This is more demanding than the corresponding requirement when using path realism (see above), but this is the price to pay for only using EPR elements of reality in the derivation of the bound. One should also note that the early-setting choice and the late-setting choice needs to be independent, which would be achieved with a good source of randomness. For the purists [15] the late-setting and early-setting choices need to be spacelike separated, something that can be achieved with independent sources of randomness suitably arranged around the interferometer.

We have found that when establishing the time of emission as an EPR element of reality, postselection still has negative effects, but only on half of the selected subensemble. This modifies the relevant Bell inequalities, but some are still violated by the quantum prediction (5). There is no local realist model with realist emission time that gives the probabilities predicted by quantum mechanics for the Franson interferometer.

TABLE I. Critical visibilities for violation of chained Bell inequalities that the outcomes from a local realist model with long realist delays (as specified in Theorem 5) must obey.

Number of terms	Emission-time realism bound	Quantum prediction	Critical visibility
4	3	$4\cos\frac{\pi}{4}\approx 2.828$	> 100%
6	5	$6\cos\frac{\pi}{6}\approx 5.196$	96.23%
8	7	$8\cos\frac{\pi}{8}\approx 7.391$	94.71%
10	9	$10\cos\frac{\pi}{10}\approx 9.511$	94.63%
12	11	$12\cos\frac{\pi}{12}\approx 11.59$	94.90%
$2N\geq 14$	$2N-1$	$2N\cos\frac{\pi}{2N}$	incr. with N

V. MODIFIED SETUPS

Another approach to reach a violation of local realism from the quantum-mechanical predictions is to modify the setup. A number of alternatives exist, and we will here briefly go through three of these alternatives.

The first was proposed by Strekalov *et al* in 1996 [16] (see Fig. 6). This setup uses a polarization-entangled source and three polarizing beamsplitters at each site. The interference occurs at the third and last polarizing beamsplitter. In this setup, the path taken *is* an EPR element of reality because a polarization measurement (in place of one interferometer) can be used to remotely predict which path the remote photon is going to travel through. But this is not needed; because of the polarization entanglement, the photon pairs are either jointly delayed or not, so that every pair gives coincident detections. There is no postselection, and we can in fact use Theorem 1 (modulo other experimental problems). This is good, because the experimental realization proposed in [16] does not use separate paths, but instead uses a single birefringent optical element at each site to implement the whole unbalanced interferometer, so an argument based on a realist path cannot be used. But as we have seen, local realism can be violated in this setup even when the paths coincide.

The second alternative uses a source proposed in Brendel *et al* in 1999 [17] and switched mirrors in place of the first beamsplitters [18]. The source uses a pulsed pump, an unbalanced interferometer, and a nonlinear device that creates photon pairs. The active mirrors are pushed in and pulled out of the photon path in sync with the source (see Fig. 7). In this setup, the path taken is also an EPR element of reality, because a measurement of the time of emission (in place of one interferometer)

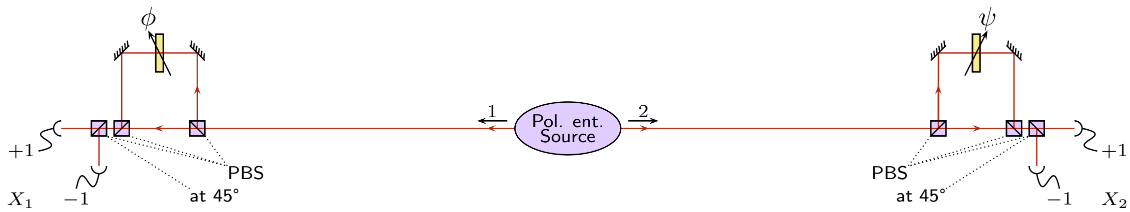

FIG. 6. Using a polarization-entangled source. The time-correlated photons are still sent out at unknown moments in time, but are now also polarization-entangled. The beamsplitters used are polarizing beamsplitters (PBS), and the interference occurs at the third PBS at each site, because this is oriented $\pi/4$ in relation to the other two. In this setup there is no postselection, so that Theorem 1 can be used directly, and the bound is violated by the quantum prediction.

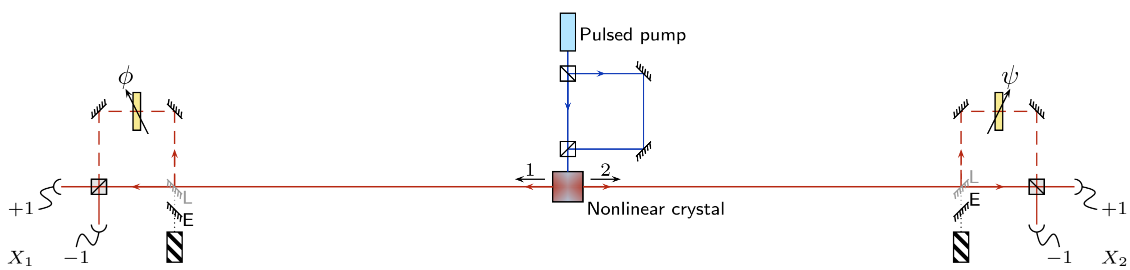

FIG. 7. Using controlled mirrors. The mirrors are synchronized with the source so that photons produced by the early part of the pulse would be delayed in the interferometer, while photons produced by the late part of the pulse would not be delayed. Also in this setup there is no postselection, so that Theorem 1 can be used directly, and the bound is violated by the quantum prediction.

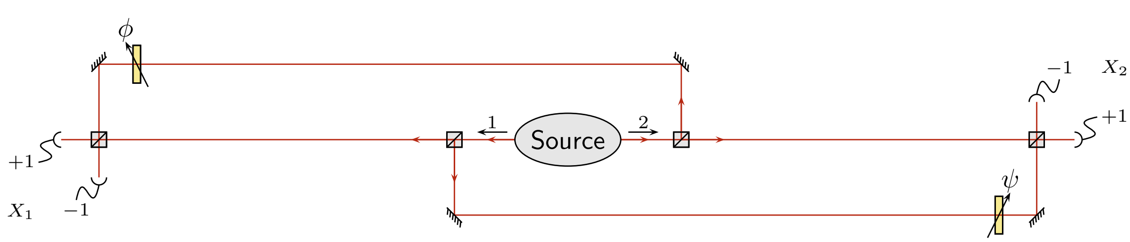

FIG. 8. Cross-coupled interferometers. This setup has many of the properties of the original Franson setup. It has very large interferometers, and this is of course experimentally challenging. Here, the path taken by each photon is an EPR element of reality so that Theorem 4 can be used, and the bound is violated by the quantum prediction.

can be used to remotely predict which path the remote photon is going to travel through. Again, this is not needed; because of the synchronization of the mirror positions, the photon pairs are either jointly delayed or not, and every pair gives coincident detections. There is no postselection, and Theorem 1 (modulo other experimental problems) can be used to rule out local realist models.

The third alternative in this list was proposed by Cabello *et al* in 2009 [19] and uses cross-coupled interferometers (see Fig. 8). In this setup, postselection is performed because the two photons may both end up at the same site, at both input ports of the beamsplitter at that site but delayed with respect to each other. The events that give coincident detection at both sites make up 50% of the total. But also here the path is an EPR element of reality, because a local path measurement (done by removing the final beamsplitter) can be used to remotely predict which path the remote photon is going to emerge from. Since the path is an EPR element of reality, we can use Theorem 4 (modulo other experimental problems), and the quantum prediction will violate the bound. There is no local realist model (with realist path) that gives the quantum predictions of this setup.

VI. CONCLUSIONS

This paper has discussed tests of local realism using energy-time entanglement. Even though most of the proposed experiments use postselection, we have seen that certain types of models can be ruled out. But we have also seen that the tests are subtle, because depending on the precise requirements made on the local realist model, the same experimental data can imply a) no violation, b) smaller violation than usual, or c) full violation of the appropriate statistical bound.

Using only that the measurement outcomes are EPR elements of reality is not enough to yield a violation [4]: the appropriate Bell inequality would be trivial and reads

$$\begin{aligned}&\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)-E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big|\leq 4.\end{aligned}\tag{13}$$

Adding that the moment of emission is an EPR element of reality enables a violation, although a weaker violation than that of the usual Bell setup [4]: there is a violation using the chained inequality (12) with six terms while the appropriate four-term Bell inequality would read

$$\begin{aligned}&\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)-E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big|\leq 3.\end{aligned}\tag{14}$$

Requiring that the model also uses a realist path will enable a violation, equally large as that from a standard Bell test [5, 8]. Note that, in the original Franson setup, the path is not an EPR element of reality; it cannot be predicted without disturbing the system. But path realism can still be listed as an expected model property, and tested in experiment; one motive for requiring this particle-like behaviour from the model would be correspondence to properties from classical physics. The violation would be equally strong as that of the usual Bell setup: the appropriate Bell inequality has the usual bound

$$\begin{aligned}&\Big|E\big(X_1(\alpha)X_2(\delta)\big|\text{coinc.}\big)+E\big(X_1(\alpha)X_2(\beta)\big|\text{coinc.}\big)\Big|\\&+\Big|E\big(X_1(\gamma)X_2(\beta)\big|\text{coinc.}\big)-E\big(X_1(\gamma)X_2(\delta)\big|\text{coinc.}\big)\Big|\leq 2.\end{aligned}\tag{15}$$

A final alternative is to modify the setup to ensure that the path taken is an EPR element of reality [16–19]. This enables a violation that is equally strong as that of the usual Bell setup: the appropriate Bell inequality is Ineq. (15).

These considerations must also be taken into account in quantum communication. For example, in Bell-inequality based quantum cryptography [20], the inequality is used as test of security. The original Bell inequality (bounded by 2) is only available as a security test if path realism can be used, and this is only possible when a) there really are distinct paths within the interferometer, and b) the attacker is unable to control which path the "photon will take." This is highly device-dependent; when aiming for device-independent security, the security test should not rely on the internal structure of the analyzing stations. Thus, path realism should not be used. The users must rely on the black-box formulation obtained when using emission time as an EPR element of reality. In this case, the original four-correlation Bell inequality cannot be used as test of security (see above), and using the chained inequalities, the security margin will be smaller than from the standard Bell test, since the critical visibility is higher than in this test.

Energy-time entanglement obtained with the Franson interferometer and its variants is a subtle way to test local realistic models, with or without added properties. It will be used in many future experiments intended to violate local realism, but in performing them it is important to be aware of exactly what is tested in these experiments, and the size of the violation. Even with the subtleties associated with this interferometer, or more correctly, because of these subtleties, the interferometer will continue to be an important tool to extend our knowledge on the foundations of quantum mechanics.

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